MENIIT
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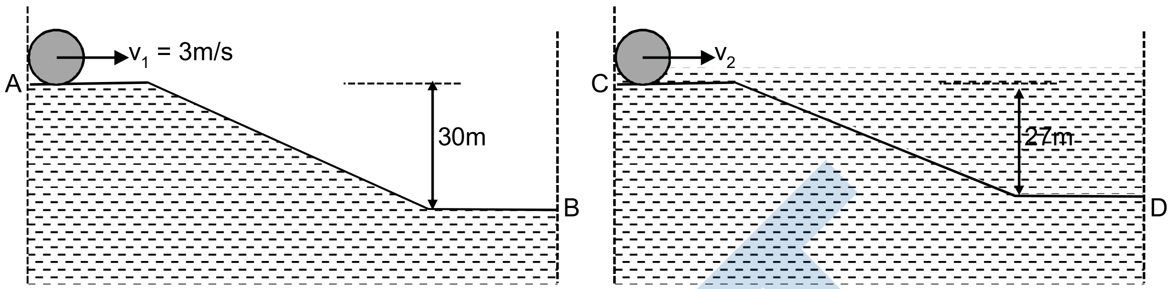
JEE Advanced : Paper-1 (2015)

IMPORTANT INSTRUCTIONS

1. The question paper has three parts: **Physics**, **Chemistry** and **Mathematics**. Each part has three sections.
2. Section 1 contains 8 questions. The answer to each question is a single digit integer ranging from 0 to 9 (both inclusive).
   Marking Scheme: +4 for correct answer and 0 in all other cases.
3. Section 2 contains 10 multiple choice questions with one or more than one correct option.
   Marking Scheme: +4 for correct answer, 0 if not attempted and –2 in all other cases.
4. Section 3 contains 2 “match the following” type questions and you will have to match entries in Column I with the entries in Column II.
   Marking Scheme: for each entry in Column I, +2 for correct answer, 0 if not attempted and – 1 in all other cases.

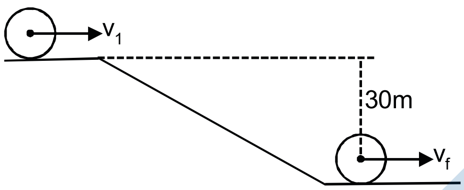
PART A: PHYSICS

1. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and v_1 = 3 m/s, then v_2 in m/s is ($g = 10\ m/s^2$)



Ans. **[7]**

Sol.



$$\frac{1}{2}mv_1^2 + \frac{1}{2}\frac{mr^2}{2}\frac{v_1^2}{r^2} + mg(30) = \frac{1}{2}mv_f^2 + \frac{1}{4}mv_f^2 = \frac{3}{4}mv_f^2$$ ⇒ for first ball

$$\frac{3}{4}mv_1^2 + mg(30) = \frac{3}{4}mv_f^2 = \frac{3}{4}mv_2^2 + mg(27)$$ ⇒ for both the balls.

$$\frac{3}{4}mv_2^2 = \frac{3}{4}mv_1^2 + 3mg \Rightarrow v_2^2 = v_1^2 + 4g = 9 + 40 = 49$$

v_2 = 7 m/sec.

2. Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A mits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is

Ans. **[2]**

Sol. $r_A = 400\ r_B$

$P_A = 10^4\ P_B$

$P_A = \sigma\ (4\pi r_A^2)\ T_A^4$

$P_B = \sigma(4\pi r_B^2)\ T_B^4$

$$\frac{P_A}{P_B} = (400)^2\left(\frac{T_A}{T_B}\right)^4 = 10^4$$

$$\left(\frac{T_A}{T_B}\right)^4 = \frac{10^4}{400 \times 400} = \frac{1}{4 \times 4} = \frac{1}{24}$$

$$\frac{T_A}{T_B} = \frac{1}{2} \qquad \text{Now } \lambda \propto \frac{1}{T}$$

$$\frac{\lambda_A}{\lambda_B} = \frac{T_B}{T_A} = 2$$

3. A nuclear power plant supplying electrical power to a village uses a radioactive material of half life T years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years, then the value of n is

Ans. **[3]**

Sol. If N_0 is total initial material available

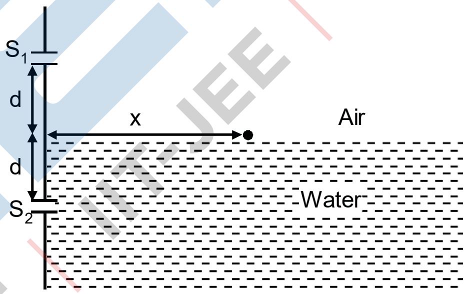
Power requirement = 12.5% of available, so plant will be able to supply till the amount becomes 12.5% of initial, so

$$N_0 \xrightarrow{T} \frac{N_0}{2} \xrightarrow{T} \frac{N_0}{4} \xrightarrow{T} \frac{N_0}{8}$$

50% 25% 12.5%

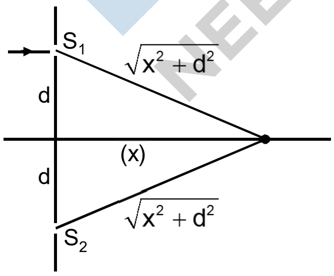
So total time = 3T

4. A Young's double slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index = 4/3) as shown in the figure. The positions of maxima on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where λ is the wavelength of light in air (refractive index = 1), 2d is the separation between the slits and m is an integer. The value of p is



Ans. **[3]**

Sol.



ΔP = path difference (optical) = $m\lambda$ = $(\mu - 1)^2 (x^2 + d^2)$

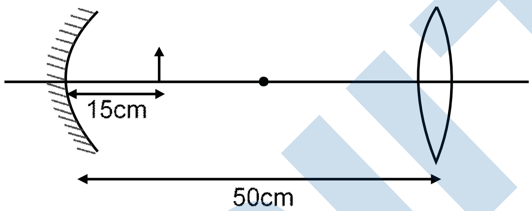
$\Rightarrow x^2 = \frac{m^2\lambda^2}{(\mu-1)^2} - d^2$

On comparing $x^2 = p^2m^2\lambda^2 - d^2$

$p^2 = \frac{1}{(\mu-1)^2} = \frac{1}{(4/3-1)^2} = \frac{1}{(1/3)^2} = 9$

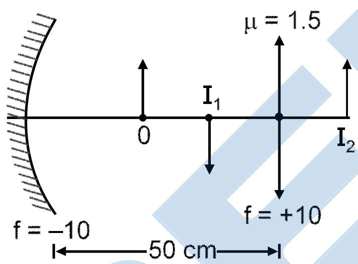
So, p = 3

5. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept in a medium of refractive index 7/6, the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is



Ans. **[7]**

Sol.



For Mirror

u = –15; f = –10

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-10} + \frac{1}{15} = \frac{-3+2}{5} = \frac{1}{-30}$

v_1 = 20 cm

For Lens

u_2 = –20 cm; f = +10

$\frac{1}{v_2} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}$

v_2 = 20 cm

$m_1 = -\frac{v_1}{u_1} = -\frac{30}{15} = -2$

$m_2 = \frac{v_2}{u_2} = \frac{20}{-20} = -1$

So, $M_1 = m_1 m_2 = +2$

In medium focal length of lens will get changed

$\frac{1}{10} = \left(\frac{3}{2}-1\right)\left\{\frac{1}{R_1}-\frac{1}{R_2}\right\} = \frac{1}{2}\left\{\frac{1}{R_1}-\frac{1}{R_2}\right\}$ $\Rightarrow$ in air

$\frac{1}{f'} = \left(\frac{3\times 6}{2\times 7}-1\right)\left\{\frac{1}{R_1}-\frac{1}{R_2}\right\} = \frac{4}{14}\left\{\frac{1}{R_1}-\frac{1}{R_2}\right\} \Rightarrow$ in medium

$\frac{f_1}{10} = \frac{1}{2}\times\frac{14}{4} = \frac{7}{4}$ so, $f_1 = \frac{7}{4}\times 10 = \frac{35}{2}$ cm

So for lens $u_2 = -20;\ f_2 = \frac{35}{2}$

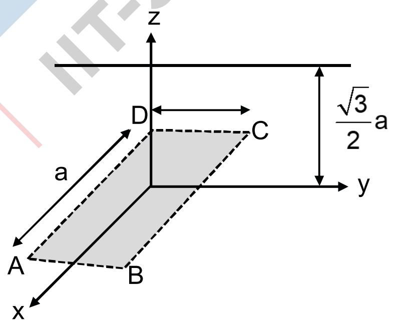
$\frac{1}{v'_2} = \frac{2}{35} - \frac{1}{20} = \frac{8-7}{140} = \frac{1}{140}$

$m_2' = \frac{v'_2}{u_2} = \frac{140}{-20} = -7$

So, $M_2 = -2 \times -7 = 14$

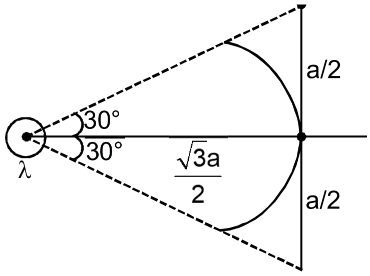
SO, $\left|\frac{M_2}{M_1}\right| = \frac{14}{2} = 7$

6. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface ABCD lying in the x-y plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is



Ans. **[6]**

Sol.



Flux through this rectangular surface will be equal to flux through curved surface (part of cylindrical surface) shown, which subtends an angle of 60° at wire.

For this cylindrical surface charge enclosed = λL

So, $\phi = \frac{\lambda L}{6\varepsilon_0}$. So, n = 6

7. Consider a hydrogen atom with its electron in the n^{th} orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of n is (hc = 1242 eV nm).

Ans. **[2]**

Sol. $KE = \left(\frac{1242}{90} - \frac{13.6}{n^2}\right) eV = 10.4\ eV$

$= 13.8\ eV - \frac{13.6}{n^2}\ eV = 10.4\ eV$

$\left(\frac{13.6}{n^2}\right) eV = (13.8 - 10.4)\ eV = 3.4\ eV$

So, n = 2

8. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is $1/4^{th}$ of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere)

Ans. **[2]**

Sol. $g = g_0 \frac{R^2}{(R+h)^2} = \frac{g_0}{4}$

$\Rightarrow 4R^2 = (R + 4)^2$

$2R = R + h \Rightarrow h = R$

$So, -\frac{GMm}{R} + \frac{1}{2}mV^2 = \frac{-GMm}{2R} + 0$

$\frac{1}{2}mV^2 = \frac{GMm}{2R} \Rightarrow V = \sqrt{\frac{GM}{R}} = \frac{V_e}{\sqrt{2}}$

$as\ Ve = \sqrt{\frac{2GM}{R}}$

So, $Ve = \sqrt{2}\ V$

N = 2

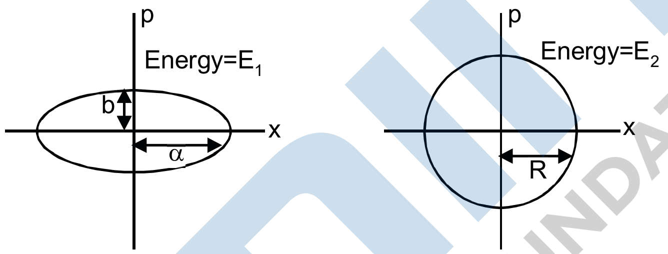
Section 2

(Maximum Marks : 40)

This section contains TEN questions.

- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- Marking Scheme:
  - + 4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
  - 0 If none of the bubbles is darkened.
  - – 2 In all other cases.

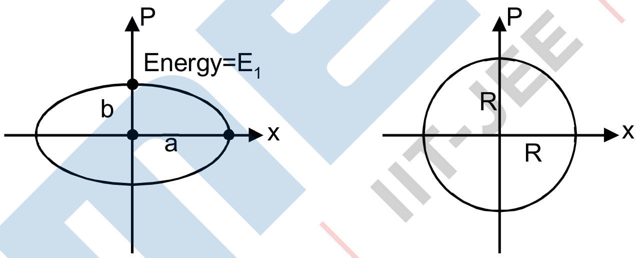
9. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with position x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equation(s) is (are) :



(A) $E_1\omega_1 = E_2\omega_2$ (B) $\frac{\omega_2}{\omega_1} = n^2$ (C) $\omega_1\omega_2 = n^2$ (D) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

Ans. **[B, D]**

Sol.



x = a; P = 0; x = 0; P = b

$b = Ma\omega_1$ .....(i) $\Rightarrow$ for 1st particle

$$E_1 = \frac{b^2}{2m}$$

$R = mR\omega_2$ ......(ii) $\Rightarrow$ for 2nd particle

$$E_2 = \frac{R^2}{2m} = \frac{ab}{2m}$$

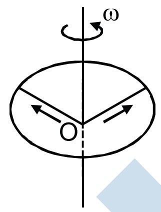
$$\frac{E_1}{E_2} = \frac{b}{a} = \frac{1}{n^2} = \frac{\omega_1}{\omega_2}$$

$m\omega_2 = 1;\ m\omega_1 = \frac{b}{a}$ (from (i) & (ii) equations)

$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{b}{a} = \frac{1}{n^2}$

$\Rightarrow R^2 = ab$

10. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O. At this instant the distance of the other mass from O is:



(A) $\frac{2}{3}R$ (B) $\frac{1}{3}R$ (C) $\frac{3}{5}R$ (D) $\frac{4}{5}R$

Ans. **[D]**

Sol. Angular momentum conservation

$$MR^2\omega = MR^2\left(\frac{8}{9}\omega\right) + \frac{M}{8}\left(\frac{3}{5}R\right)^2\frac{8}{9}\omega + \frac{M}{8}x^2\frac{8}{9}\omega$$

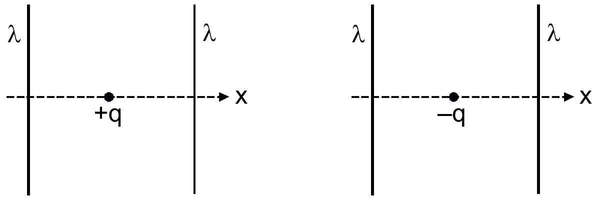
$$R^2 = \frac{8}{9}R^2 + \frac{9}{25}\times\frac{1}{9}R^2 + \frac{1}{9}x^2$$

$$= \left(\frac{8}{9} + \frac{1}{25}\right)R^2 + \frac{1}{9}x^2 = \left(\frac{200+19}{225}\right)R^2 + \frac{1}{9}x^2$$

$$R^2\left\{1 - \frac{209}{225}\right\} = \frac{1}{9}x^2$$

$$x = \frac{3\times4}{15}R = \frac{4R}{5}$$

11. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and – q are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then correct statement(s) is(are)



(A) Both charges execute simple harmonic motion

(B) Both charges will continue moving in the direction of their displacement

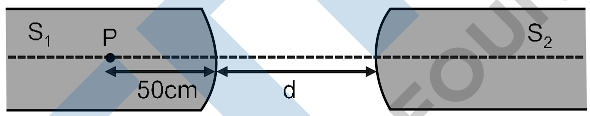
(C) Charge +q executes simple harmonic motion while charge –q continues moving in the direction of its displacement

(D) Charge – q executes simple harmonic motion while charge +q continues moving in the direction of its displacement.

Ans. [C]

Sol. +q charge is in position of stable equilibrium, so for small oscillations it will perform SHM, –q charge will move in +ve direction only on displacing it in +ve x.

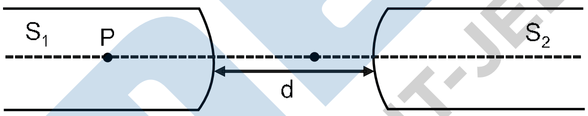
12. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is



(A) 60 cm (B) 70 cm (C) 80 cm (D) 90 cm

Ans. [B]

Sol.



$u_1 = -50$ cm

$n_1 = 1.5$

$n_2 = 1$

$R = -10$

So, $\frac{1}{v_1} + \frac{3}{2 \times 50} = \frac{1-1.5}{-10} = \frac{1}{20}$

$\frac{1}{v_1} = +\frac{1}{20} - \frac{3}{100} = \frac{+5-3}{100}$

$v_1 = 50$ cm

For S_2

$u = -(d-50)$; $n_1 = 1$; $n_2 = 1.5$; $v = \infty$; $R = +10$

$$\frac{3}{2(\infty)} + \frac{1}{(d-50)} = \frac{1}{2\times 10}$$

$d - 50 = 20$

So, $d = 70$ cm

If

$u = +(50 - d)$

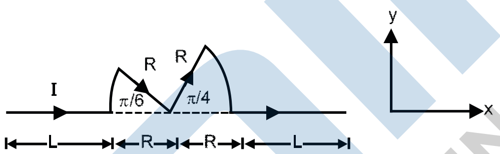
$n_1 = 1$; $n_2 = 1.5$; $v = \infty$; $R = +0$

$$\frac{3}{2(\infty)} + \frac{1}{50-d} = \frac{1}{20}$$

So, $50 - d = 20$

$d = 50 - 20 = 30$ cm

13. A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statement(s) is (are)



(A) If $\vec{B}$ is along $\hat{z}$, $F \propto (L + R)$

(B) If $\vec{B}$ is along $\hat{x}$, $F = 0$

(A) If $\vec{B}$ is along $\hat{y}$, $F \propto (L + R)$

(B) If $\vec{B}$ is along $\hat{z}$, $F = 0$

Ans. **[A, B, C]**

Sol. In uniform $\vec{B}$, random shaped conductor can also be replaced by a straight conductor of length

$= L + 2R + L = 2(R + L)\ \hat{i}$

(A) $\vec{F} = i\vec{\ell}\times\vec{B} = 2i\ (R + L)\ ((\hat{i}\times\hat{k})B_0 = 2iB0\ (R + L)(-\hat{j})$, $F \propto (R + L)$

(B) $\vec{F} = 0$ if $\vec{B} = B_0\hat{i}$ as $= \hat{\ell}\times\vec{B} = 0$

(C) $\vec{F} = 2i(R+L)\left(\hat{i}\times\hat{j}\right)B0 = 2iB0 + (R + L)\ \hat{k}$, $F \propto (R + L)$

(D) $\vec{F} = 2iB_0(R+L)(\hat{i}\times\hat{k}) \neq 0$

14. A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is (are)

(A) The average energy per mole of the gas mixture is 2RT

(B) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$

(C) The ratio of the rms speed of helium atoms to that of hydrogen molecules is 1/2

(D) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$

Ans. **[A, B, D]**

Sol. Average energy per mole of mixture

$$=\frac{\left(\frac{3}{2}RT+\frac{5}{2}RT\right)}{2}=\frac{8RT}{4}=2RT$$

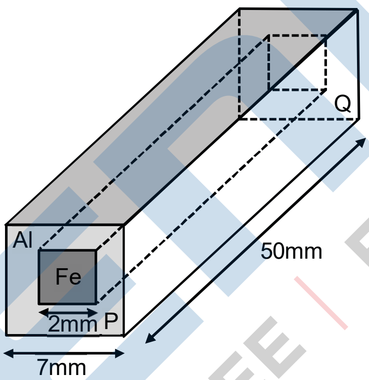
$$C_{v,mix}=\left(\frac{3}{2}R+\frac{5}{2}R\right)\frac{1}{2}=2R$$

$$C_{P,mix}=\left(\frac{5}{2}R+\frac{7}{2}R\right)\frac{1}{2}=3R$$

$$\lambda_{mix}=\frac{3}{2};\ Y_{He}=\frac{5}{3}$$

$$\text{So, }\frac{V_{mix}}{V_{He}}=\sqrt{\frac{Y_{mix}RT}{M_{mix}}\cdot\frac{M_{He}}{Y_{He}RT}}=\sqrt{\frac{3}{2}\cdot\frac{4}{3}\cdot\frac{3}{5}}=\frac{V_{rms,\,He}}{V_{rms,\,H_2}}=\sqrt{\frac{M_{H_2}}{M_{He}}}=\sqrt{\frac{2}{4}}=\frac{1}{\sqrt{2}}$$

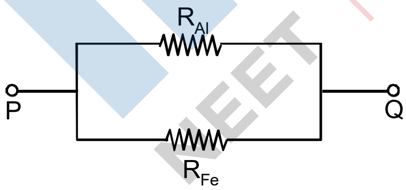
15. In an aluminum (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are $2.7 \times 10^{-8}\ \Omega$ m and $1.0 \times 10^{-7}\ \Omega$ m, respectively. The electrical resistance between the two faces P and Q of the composite bar is



(A) $\frac{2475}{64}\mu\Omega$ (B) $\frac{1875}{64}\mu\Omega$ (C) $\frac{1875}{49}\mu\Omega$ (D) $\frac{2475}{132}\mu\Omega$

Ans. **[B]**

Sol.

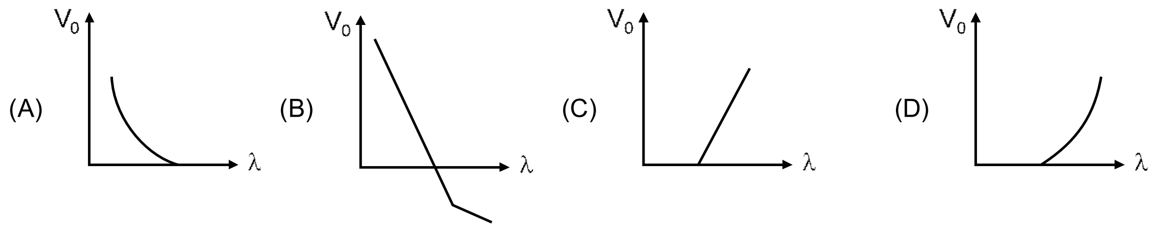


$$R_{Al} = 2.7 \times 10^{-8} \times \frac{50}{45\times10^{-3}}=\frac{27\times5}{45}\times 10^{-5} = 30\mu\Omega$$

$$R_{Fe} = 1.0 \times 10^{-7} \times \frac{50}{45\times10^{-3}}=\frac{50}{4}\times10^{-4}=\frac{5000}{4}\ \mu\Omega = 1250\ \mu\Omega$$

$$R_g=\frac{1250\times30}{1280}\mu\Omega=\frac{625\times3}{64}\mu\Omega=\frac{1875}{64}\mu\Omega$$

16. For photo-electric effect with incident photon wavelength λ, the stopping potential is V_0. Identify the correct variation(s) of V_0 with λ and $\frac{1}{\lambda}$.



Ans. **[A, C]**

Sol. $ev_0 = \frac{hc}{\lambda} - \phi$

$v_0 = \left(\frac{hc}{e}\right)\frac{1}{\lambda} = \frac{\phi}{e}$

v_0 vs $\frac{1}{\lambda}$ graph will be as shown in (C) and for v_0 vs λ graph will be as shown in (A)

17. Consider a Vernier callipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Vernier callipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then:

(A) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.

(B) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.

(C) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.

(D) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.

Ans. **[B, C]**

Sol. $1\text{ MSD} = \left(\frac{1\text{cm}}{8}\right)$

$5\text{VSD} = 4\text{ MSD} = \frac{4}{8}\text{cm} = \frac{1}{2}\text{cm}$

$1\text{ VSD} = \frac{1}{10}\text{ cm} = 1\text{ mm}$

So, $LC = 1\text{ MSD} - 1\text{ VSD} = \left(\frac{10}{8} - 1\right)\text{ mm} = \left(\frac{5}{4} - 1\right)\text{ mm} = \frac{1}{4}\text{mm} = 0.25\text{ mm}$

For Screw Gauge

(B) Pitch = 2 × 0.25 mm = 0.5 mm

100 circular parts = pitch = 0.5 mm

So, 1 circular part = $\frac{0.5}{100}$ mm = 0.005 mm

(C) LC of linear scale of screw gauge = 2 × 0.25 mm = 0.5 mm

100 circular parts = 2 × 0.5 mm = 1 mm

So 1 circular division = 0.01 mm

18. Planck's constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. Then the correct option(s) is(are)

(A) $M \propto \sqrt{c}$ (B) $M \propto \sqrt{G}$ (C) $L \propto \sqrt{h}$ (D) $L \propto \sqrt{G}$

Ans. **[A, C, D]**

Sol. $h = \frac{E}{\nu} = \frac{ML^2T^{-2}}{T^{-1}} = ML^2T^{-1}$

$c = LT^{-1}$

$G = \frac{Fr^2}{m_1m_2} = \frac{ML^3T^{-2}}{M^2} = M^{-1}L^3T^{-2}$

$hc = ML^3T^{-2}$; $G = M^{-1}L^3T^{-2}$

$\Rightarrow \frac{hc}{G} = M^2 \Rightarrow M = \sqrt{\frac{hc}{G}}$

$\frac{h}{c} = ML$ So, $L = \frac{h}{c}\sqrt{\frac{G}{hc}} = \sqrt{\frac{Gh}{c^3}}$

$M \propto \sqrt{c}$

$L \propto \sqrt{h}$

$L \propto \sqrt{G}$

Section 3

(Maximum Marks : 16)

- This section contains TWO questions.
- Each question contains two columns, Column-I and Column-II
- Column-I has four entries (A), (B), (C) and (D)
- Column-II has five entries (P), (Q), (R), (S) and (T)
- Match the entries in Column-I with the entries in Column-II
- One or more entries in Column-I may match with one or more entries in Column-II
- The ORS contains a 4 × 5 matrix whose layout will be similar to the one shown below:

(A)	P	Q	R	S	T
(B)	P	Q	R	S	T
(C)	P	Q	R	S	T

(D) [P] [Q] [R] [S] [T]

· For each entry in Column-I, darken the bubbles of all matching entries. For example, if entry (A) in Column-I matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).

· Marking Scheme:

For each entry in column I

+ 2 If only the bubble(s) corresponding to all the correct match(es) is(are) darkened.

0 If none of the bubbles is darkened.

– 1 In all other cases.

19. Match the nuclear processes given in Column I with the appropriate option(s) in Column II.

Column I	**Column II**
(A) Nuclear fusion	(P) Absorption of thermal neutrons by $^{235}_{92}U$
(B) Fission in a nuclear reactor	(Q) $^{60}_{27}Co$ nucleus
(C) β-decay	(R) Energy production in stars via hydrogen conversion to helium
(D) γ-ray emission	(S) Heavy water
	(T) Neutrino emission.

Ans. [(A) R or R, T (B) P, S (C) Q, T (D) R

Sol. Nuclear fusion - Energy production in stars via hydrogen conversion to He.

- In nuclear fusion reaction emission of neutrinos is very common.

Nuclear fission - Involves absorption of thermal neutrons.

- Heavy water is used as moderator.

- In nuclear fission reaction emission of neutrinos is very common.

β-decay- Cobalt-60 is used as source of β-particle

- Neutrino's are also released with β-particle.

γ-ray emission - After absorpsion of thermal neutrons emission of γ-radiations may take place.

- Co^{60} also involves γ-radiation.

- γ-emission is common in both fission and fusion reaction.

20. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 are constants). Match the potential energies in column I to the corresponding statement (s) in column II.

Column I	**Column II**
(A) $U_1(x) = \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	(P) The force acting on the particle is zero at x = a.
(B) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(Q) The force acting on the particle is zero at x = 0.

(C) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$

(D) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$

(R) The force acting on the particle is zero at x = – a.

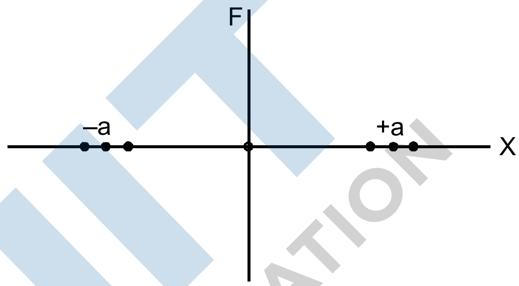
(S) The particle experiences an attractive force towards x = 0 in the region | x | < a.

(T) The particle with total energy $\frac{U_0}{4}$ can oscillate about the point x = – a.

Ans. (A) P, Q, R, T (B) Q, S (C) P, Q, R, S (D) P, R, T

Sol. (A) $U_1 = \frac{U_0}{2}\left[1 - \frac{x^2}{a^2}\right]^2$

$$F = -\frac{dU}{dx} = +\frac{U_0}{0} \times 2\left(1 - \frac{x^2}{a^2}\right)\left(1 + \frac{2x}{a^2}\right)$$



at x = a, F = 0

at x = 0, F = 0

at x = –a, F = 0

at x = –a position of stable equilibrium

x = +a position of stable equilibrium

(B) $U_2(x) = \frac{U_0}{2}\left(\frac{x^2}{a}\right)$

$$F_2 = -\frac{dU}{dx} = -\frac{2U_0 x}{2a} = \frac{-U_0}{a}x$$

(C) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$

$$F_3(x) = \frac{-dU}{dx} = -\frac{U_0}{2}\frac{2x}{a}e^{-x^2/a^2} + \frac{U_0}{2}\frac{x^2}{a}e^{-x^2/a^2}\left(\frac{2x}{a^2}\right)$$

$$= -\frac{U_0}{2}\frac{2x}{a}e^{-x^2/a^2}\left\{1 - \frac{x^2}{a^2}\right\}$$

x = –a, a

will be positions of unstable equilibrium.

(D) $U_4(x) \frac{U_0}{2}\left\{\frac{x}{a} - \frac{x^3}{3a^3}\right\}$

$$F = \frac{-dU}{dx} = -\frac{U_0}{2a} + \frac{U_0}{6a^3}3x^2 = -\frac{U_0}{2a}\left(1 - \frac{x^2}{a^2}\right)$$

x = –a ⇒ position of stable equilibrium

x = +a ⇒ position of unstable equilibrium

PART B: CHEMISTRY

Section 1

(Maximum Marks : 32)

- This section contains Eight question.
- The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- Marking Scheme

  + 4 If the bubble corresponding to the answer is darkened.

  0 In all other cases.

21. For the octahedral complexes of Fe^{3+} in SCN^- (thiocyanato - S) and in CN^- ligand environments, the difference between the spin only magnetic moments in Bohr magnetons

(when approximated to the nearest integer) is:

[Atomic number of Fe = 26]

Ans. **[4]**

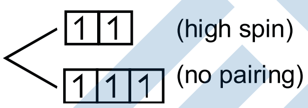
Sol. $[Fe(SCN)_6]^{3-}$ and $[Fe(CN)_6]^{3-}$

In both the cases the electronic configuration of Fe^{3+} will be

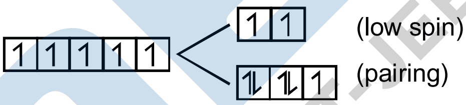
$1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^5$

Since SCN is a weak field ligand and $\bar{C}N$ is a strong field ligand, the pairing will occur only in case of $[Fe(CN)_6]^{3-}$

Case -1 $3d^5$



Case -2



$$\text{Case}-1\ \mu=\sqrt{n(n+2)}=\sqrt{5(5+2)}=\sqrt{35}=5.91\text{BM}$$

$$\text{Case}-2\ \mu=\sqrt{n(n+2)}=\sqrt{1(1+2)}=\sqrt{3}=1.73\text{ BM}$$

Difference in spin only magnetic moment = 5.91 – 1.73 = 4.18 ≈ 4

22. Among the triatomic molecules / ions, $BeCl_2$, N_3^-, N_2O, NO_2^+, O_3, SCl_2, ICl_2^-, I_3^- and XeF_2 the total number of linear molecule(s) / ion(s) where the hybridization of the central atom does not have contribution form the d-orbital(s) is:

[Atomic number: S = 16, Cl = 17, I = 53 and Xe = 54]

Ans. **[4]**

Sol. $BeCl_2, N_3^-, N_2O, NO_2^+, O_3, SCl_2, ICl_2^-, I_3^-, XeF_2$

$BeCl_2 \longrightarrow sp \longrightarrow$ linear

$N_3^- \longrightarrow sp \longrightarrow$ linear

$N_2O \longrightarrow sp \longrightarrow$ linear

$\overset{\oplus}{N}O_2 \longrightarrow sp \longrightarrow$ linear

$O_3 \longrightarrow sp^2 \longrightarrow$ bent

$SCl_2 \longrightarrow sp^3 \longrightarrow$ bent

$I_3^- \longrightarrow sp^3d \longrightarrow$ linear

$ICl_2^- \longrightarrow sp^3d \longrightarrow$ linear

$XeF_2 \longrightarrow sp^3d \longrightarrow$ linear

So among the following only four (4) has linear shape and no d-orbital is involved in hybridization

23. Not considering the electronic spin, the degeneracy of the second excited state (n = 3) of H atom is 9, while the degeneracy of the second excited state of H^- is:

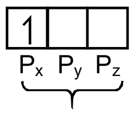
Ans. **[3]**

Sol. single electron species don't follow the $(n + \ell)$ rule but multi electron species do.

Ground state of $H^- = 1s^2$

First excited state of $H^- = 1s^1, 2s^1$

Second excited state of $H^- = 1s^1, 2s^0, 2p^1$



(3 degenerate orbitals)

24. All the energy released from the reaction $X \rightarrow Y$, $\Delta_r G^0 = -193$ kJ mol^{-1}

is used form oxidizing M^+ as $M^+ \rightarrow M^{3+} + 2e^-$, $E^0 = -0.25$ V.

Under standard conditions, the number of moles of M^+ oxidized when one mole of X is converted to Y is: [F = 96500 C mol^{-1}]

Ans. **[4]**

Sol. $X \longrightarrow Y$, $\Delta_r G^0 = 193$ KJ/ mol

$M^+ \longrightarrow M^{3+} + 2e^-$ $E^0 = 0.25$V

ΔG^0 for the this reaction is

$\Delta G^0 = -FE^0 = -2\times*(-0.25)\times 96500 = 48250$ J/mol

48.25 kJ/mole

So the number of M^+ oxidized using $X \longrightarrow Y$ will be $= \frac{193}{48.25} = 4$ moles

25. If the freezing point of a 0.01 molal aqueous solution of a cobalt (III) chloride - ammonia complex (which behaves as a strong electrolyte) is – 0.0558°C, the number of chloride(s) in the coordination sphere of the complex is:

[K_f of water = 1.86 K kg mol^{-1}]

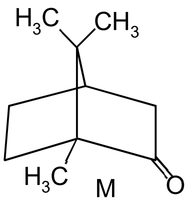
Ans. [1]

Sol. $\Delta T_f = iK_f m$

$0.0558 = i \times 1.86 \times 0.01$

$i = 3$

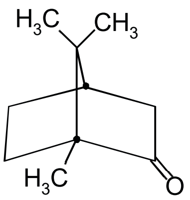
26. The total number of stereoisomers that can exist for **M** is:



M

Ans. [2]

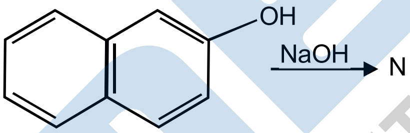
Sol.



Bridging does not allow the other 2 variants to exist.

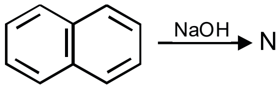
Total no. of stereoisomers of M = 2

27. The number of resonance structures for **N** is:

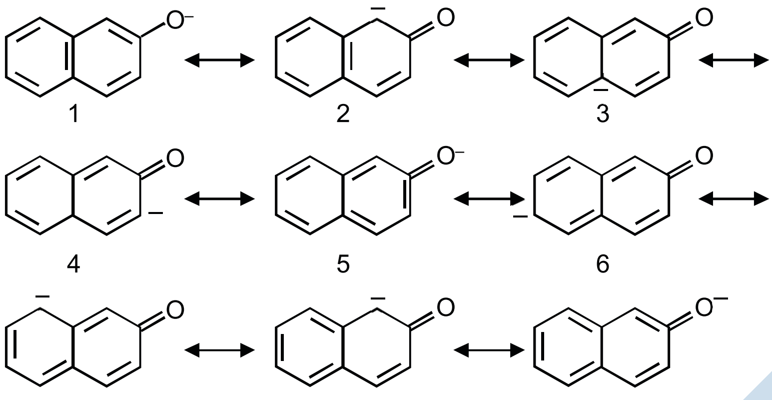


Ans. [9]

Sol.



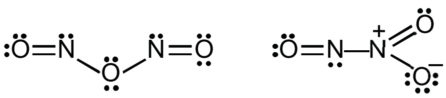
N is



28. The total number of lone pairs of electrons in N_2O_3 is:

Ans. **[8]**

Sol.



Total no. of lone pair = 8

Section 2

. This section contains TEN questions

. Each question has FOUR options (A). (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) current.

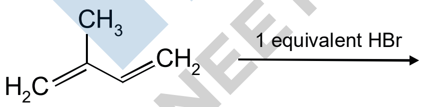
. For each question, darken the bubble(s) corresponding to the correct option(s) in the ORS Marking scheme:

+4 if only the bubble(s) corresponding to all the correct option(s) is (are) darkened

If none of the bubbles is darkened

-2 In all other cases

29. In the following reaction, the major product is:

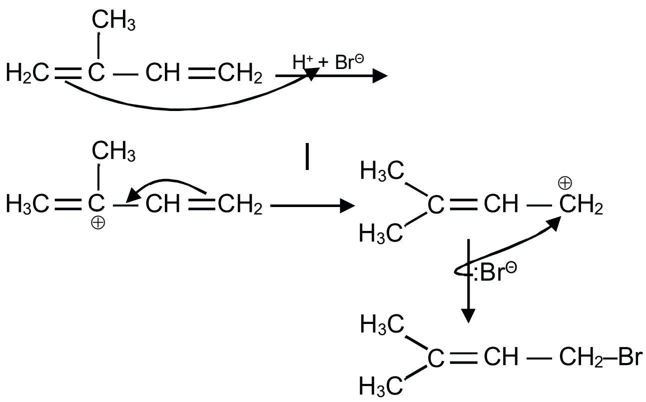


(A) 

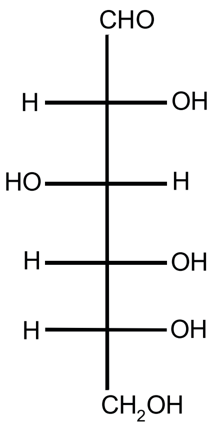
(C) 

Ans **[D]**

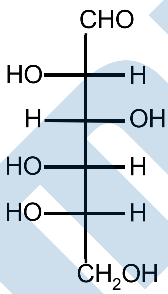
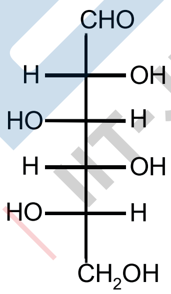
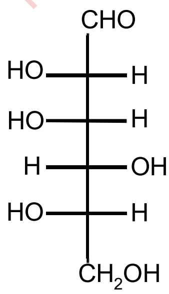
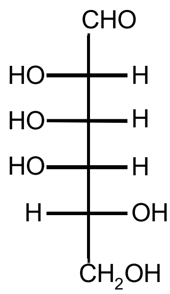
Sol.



30. The structure of D- (+) glucose is:

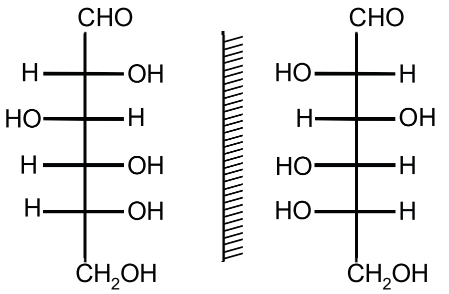


The structure of L-(–) glucose is :

(A)  (B)  (C)  (D) 

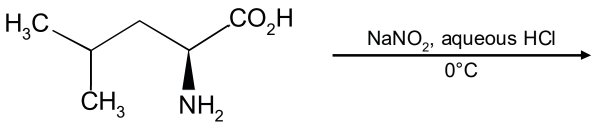
Ans. **[A]**

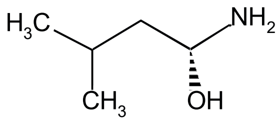
Sol.

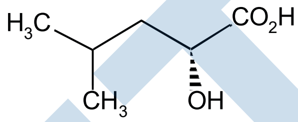


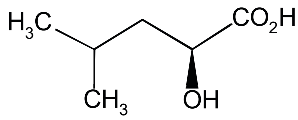
D(+) glucose L(+) glucose

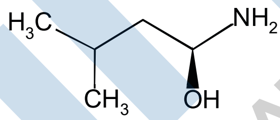
31. The major product of the reaction is:



(A)


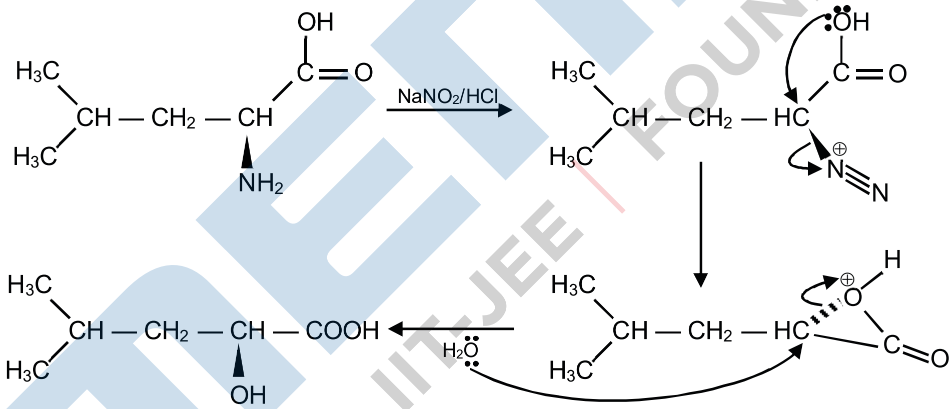
(B)


(C)


(D)


Ans. **[C]**

Sol.



32. The correct statement(s) about Cr^{2+} and Mn^{3+} is (are)

[Atomic numbers of Cr = 24 and Mn = 25]

(A) Cr^{2+} is a reducing agent

(B) Mn^{3+} is an oxidizing agent

(C) Both Cr^{2+} and Mn^{3+} exhibit d^4 electronic configuration

(D) When Cr^{2+} is used as a reducing agent, the chromium ion attains d^5 electronic configuration.

Ans. **[A], [B], [C]**

Sol. (1) Cr^{2+} is a reducing agent because Cr^{3+} is more stable.

(2) Mn^{3+} is an oxidizing agent because Mn^{2+} is more stable.

(3) Cr^{2+} and Mn^{3+} exhibit d^4 electronic configuration.

33. Copper is purified by electrolytic refining of blister copper. The correct statement(s) about this process is(are):

(A) Impure Cu strip is used as cathode

(B) Acidified aqueous $CuSO_4$ is used as electrolyte

(C) Pure Cu deposits at cathode

(D) Impurities settle as anode - mud

Ans **[B], [C], [D]**

Sol. (1) Impure Cu strip is used as anode and impurities settle as anode mud.

(2) Pure Cu deposits at cathode.

(3) Acidified aqueous $CuSO_4$ is used as electrolyte.

34. Fe^{3+} is reduced to Fe^{2+} by using

(A) H_2O_2 in presence of NaOH

(B) Na_2O_2 in water

(C) H_2O_2 in presence of H_2SO_4

(D) Na_2O_2 in presence of H_2SO_4

Ans. **[A], [B]**

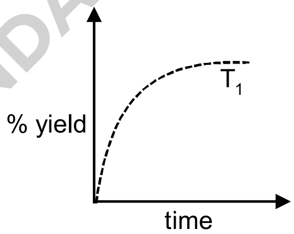
Sol. $2Fe^{+3} + H_2O_2 + 2OH^- \longrightarrow 2Fe^{+2} + 2H_2O + O_2$

$Na_2O_2 + H_2O \longrightarrow H_2O_2 + NaOH$

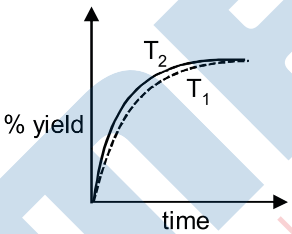
35. The % yield of ammonia as a function of time in the reaction

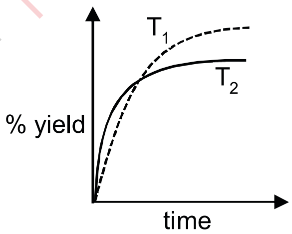
$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g), \Delta H < 0$

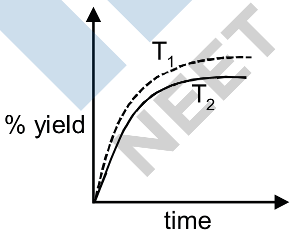
at (P, T_1) is given below:

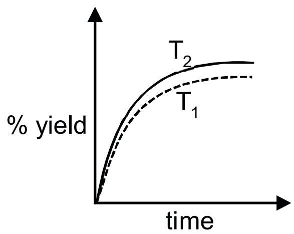


If this reaction is conducted at (P, T_2), with $T_2 > T_1$, the % yield of ammonia as a function of time is represented by :

(A) 

(B) 

(C) 

(D) 

Ans. **[B]**

Sol. $N_2(g) + 3H_2(g)$

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)\ ;\ \Delta H < 0$

36. If the unit cell of a mineral has cubic close packed (ccp) array of oxygen atoms with m fraction of octahedral holes occupied by aluminium ions and n fraction of tetrahedral holes occupied by magnesium ions, m and n, respectively, are

(A) $\frac{1}{2}, \frac{1}{8}$ (B) $1, \frac{1}{4}$ (C) $\frac{1}{2}, \frac{1}{2}$ (D) $\frac{1}{4}, \frac{1}{8}$

Ans. **[A]**

Sol. In ccp lattice:

Number of O atoms $\longrightarrow 4$

Number of Octahedral voids $\longrightarrow 4$

Number of tetrahedral voids $\longrightarrow 8$

Number of Al^{3+} = 4 × m

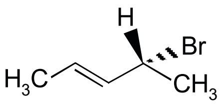
Number of Mg^{2+} = 8 × n

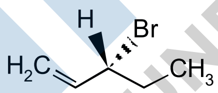
Due to charge neutrality = 0

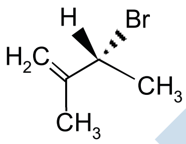
4(-2) + 4m (+3) + 8n (+2) = 0

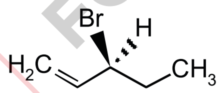
$\therefore m = \frac{1}{2} \text{ and } n = \frac{1}{8}$

37. Compound(s) that on hydrogenation produce(s) optically inactive compound(s) is(are)

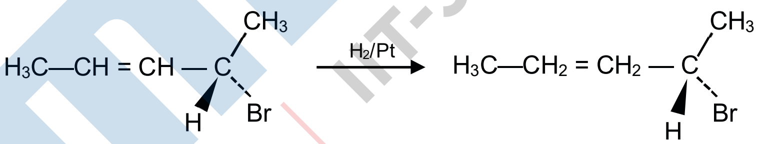
(A) 

(B) 

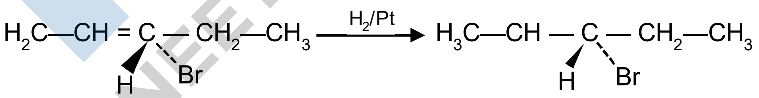
(C) 

(D) 

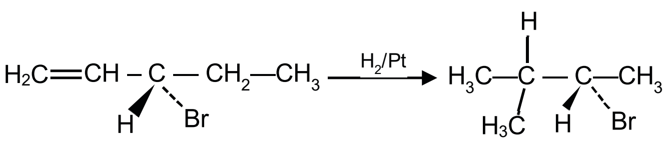
Ans. **[B], [D]**

Sol. (1) 

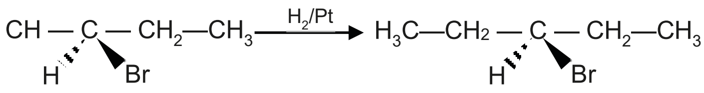
Optically active

(2) 

Optically inactive

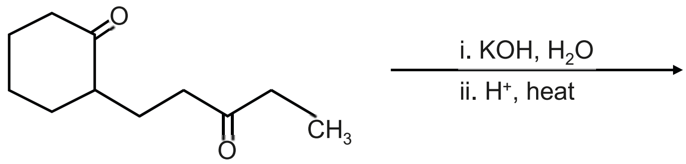
(3) 

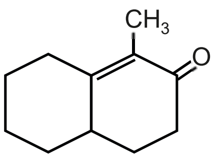
Opticallyactive

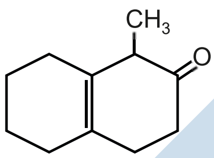
(4) 

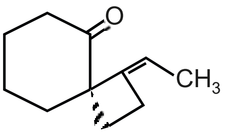
Optically inactive

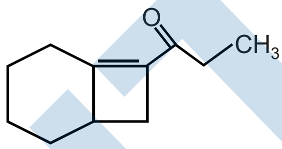
38. The major product of the following reaction is



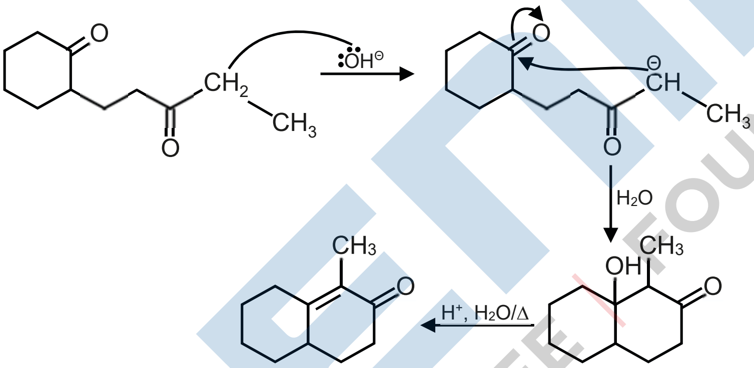
(A) 

(B) 

(C) 

(D) 

Ans. **[A]**

Sol. 

Section 3

(Maximum Marks : 16)

This section contains Two questions.

- Each question contains two columns, Column I and Column II
- Column I has four entries (A), (B), (C) and (D)
- Column II has five entries (P), (Q), (R), (S) and (T)
- Match the entries in Column I with the entries in Column II
- One or more entries in Column I may match with one or more entries in Column II
- The ORS contains a 4 × 5 matrix whose layout will be similar to the one shown below:

(A)	P	Q	R	S	T
(B)	P	Q	R	S	T

(C) [P] [Q] [R] [S] [T]

(D) [P] [Q] [R] [S] [T]

. For each entry in Column I, darken the bubbles of all the matching entries. For example, if entry (A) in Column I, matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).

. Marking scheme:

For each entry in Column I

+2 If only the bubble(s) corresponding to all the correct match(es) is(are) darkened

0 If none of the bubbles is darkened

–1 In all other cases

39. Match the anionic species given in **Column I** that are present in the ore(s) given in **Column II**

	Column I		**Column II**
(A)	Carbonate	(P)	Siderite
(B)	Sulphide	(Q)	Malachite
(C)	Hydroxide	(R)	Bauxite
(D)	Oxide	(S)	Calamine
		(T)	Argentite

Ans. **(A) PQS (B) T (C) QR (D) R**

Sol.

Siderite	$FeCO_3$
Malachite	$CuCO_3.Cu(OH)_2$
Bauxite	$AlO_x(OH)_{3-2x}$; $0 < x < 1$
Calamine	$ZnCO_3$
Argentite	Ag_2S

40. Match the thermodynamic processes given under **Column I** with the expressions given under **Column II**.

	Column I		**Column II**
(A)	Freezing of water at 273 K and 1 atm	(P)	$q = 0$
(B)	Expansion of 1 mol of an ideal gas into a vacuum under isolated conditions	(Q)	$w = 0$
(C)	Mixing of equal volumes of two ideal gases at constant temperature and pressure is an isolated container	(R)	$\Delta S_{sys} < 0$
(D)	Reversible heating of $H_2(g)$ at 1 atm from 300 K to 600 K, followed by reversible cooling to 300 K at 1 atm.	(S)	$\Delta U = 0$
		(T)	$\Delta G = 0$

Ans. **(A) RT (B) PQS (C) PQS (D) PQST**

PART C: MATHEMATICS

Section 1

(Maximum Marks : 32)

- This section contains Eight questions.
- The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- Marking Scheme

  + 4 If the bubble corresponding to the answer is darkened.

  0 In all other cases.

41. Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is

Ans. **[5]**

Sol. n = number of ways in which all girls are consecutive = 6! · 5!

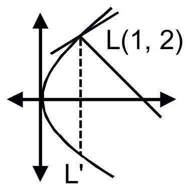
m = number of ways in which exactly 4 girls are consecutive = $^5C_4 \times 5! \times {}^6C_2 \times 2! \times 4!$

$$\therefore \; = \frac{m}{n} = \frac{^5C_4 \cdot 5! \cdot {}^6C_2 \cdot 2! \cdot 4!}{6! \cdot 5!} = 5.$$ **Ans.**

42. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x - 3)^2 + (y + 2)^2 = r^2$, then the value of r^2 is

Ans. **[2]**

Sol. Equation of normal at (1, 2) on $y^2 = 4x$ will be $y - y_1 = \frac{-y_1}{2a}(x - x_1)$



$$\Rightarrow y - 2 = \frac{-2}{2 \cdot 1}(x - 1)$$

$\Rightarrow y - 2 = -x + 1 \Rightarrow x + y = 3$

$\because$ It is tangent to $(x - 3)^2 + (y + 2)^2 = r^2$.

$\therefore$ Perpendicular distance from centre (3, – 2) = radius

$$\Rightarrow \frac{|3 - 2 - 3|}{\sqrt{2}} = r \Rightarrow r = \sqrt{2} \Rightarrow r^2 = 2.$$ **Ans.**

43. Let f: R → R be a function defined by $f(x) = \begin{cases} [x], & x \le 2 \\ 0, & x > 2 \end{cases}$, where [x] is the greatest integer less than or equal to x. If $I = \int_{-1}^{2} \frac{x\, f(x^2)}{2 + f(x+1)} dx$, , then the value of (4I – 1) is

Ans. **[0]**

Sol. $f(x) = \begin{cases} [x], & x \le 2 \\ 0, & x > 2 \end{cases} = \begin{cases} 0, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \\ 2, & x = 2 \\ 0, & x > 2 \end{cases}$

$$\therefore f(x^2) = \begin{cases} 0, & 0 \le x^2 < 1 \\ 1, & 1 \le x^2 < 2 \\ 2, & x^2 = 2 \\ 0, & x^2 > 2 \end{cases}$$

$$f(x + 1) = \begin{cases} 0, & 0 \le x+1 < 1 \\ 1, & 1 \le x+1 < 2 \\ 2, & x+1 = 2 \\ 0, & x+1 > 2 \end{cases}$$

$$I = \int_{-1}^{2} \frac{xf(x^2)}{2+f(x+1)} dx = \int_{-1}^{1} 0\, dx + \int_{1}^{\sqrt{2}} \frac{x \cdot 1}{2+0} dx + \int_{\sqrt{2}}^{2} 0\, dx = \left(\frac{x^2}{4}\right)_1^{\sqrt{2}} = \left(\frac{2-1}{4}\right) = \frac{1}{4}.$$

$\therefore$ 4I – 1 = 0. **Ans.**

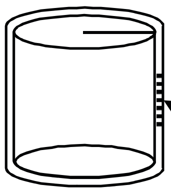
44. A cylindrical container is to be made from certain solid material with the following constraints: It has a fixed inner volume of V mm^3, has a 2 mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2 mm and is of radius equal to the outer radius of the container. If the volume of the material used to make the container is minimum when the inner radius of the container is 10 mm, then the value of $\frac{V}{250\pi}$ is

Ans. **[4]**

Sol. $V_m = \pi r_E^2 h_E - \pi r_I^2 h_I = \pi (r + 2)^2 (h + 2) - V = \pi (r + 2)^2 \left(\frac{V}{\pi r^2} + 2\right) - V$

$$\frac{dV_m}{dr} = \pi \cdot 2 (r + 2) \left(\frac{V}{\pi r^2} + 2\right) + \pi (r + 2)^2 \left(\frac{-2V}{\pi r^3}\right) = 0,$$

$$\Rightarrow \frac{2(V + 2\pi r^2)}{\pi r^2} = \frac{2V(r+2)}{\pi r^3}$$



$\Rightarrow 2Vr + 4\pi r^3 = 2Vr + 4V$

$\Rightarrow V = \pi r^3 = 1000\pi$

$\therefore \frac{V}{250\pi} = 4.$ **Ans.**

45. Let $F(x) = \int_x^{x^2+\frac{\pi}{6}} 2\cos^2 t\, dt$ for all $x \in R$ and $f : \left[0, \frac{1}{2}\right] \to [0, \infty)$ be a continuous function. For $a \in \left[0, \frac{1}{2}\right]$, if F ' (a) + 2 is the area of the region bounded by x = 0, y = 0, y = f (x) and x = a, then f (0) is

Ans. **[3]**

Sol. $F'(x) = 2\cos^2\left(x^2+\frac{\pi}{6}\right)\cdot 2x - 2\cos^2 x$

$F''(x) = \cos^2\left(x^2+\frac{\pi}{6}\right)+\left(-8x\cos\left(x^2+\frac{\pi}{6}\right)\sin\left(x^2+\frac{\pi}{6}\right)\cdot 2x\right)-2\cos^2 x$

$F''(0) = 4\cdot\left(\frac{\sqrt{3}}{2}\right) = 3^2$

$\int_0^{\alpha} f(x)\,dx = F'(\alpha) + 2$

$f(\alpha) = F''(\alpha)$

$f(0) = F''(0) = 3$. **Ans.**

46. The number of distinct solutions of the equation $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is

Ans. **[8]**

Sol. $\frac{5}{4}\cos^2 2x + 1 - 2\sin^2 x\cos^2 x + 1 - 3\sin^2 x\cos^2 x = 2$

$\Rightarrow 5\cos^2 2x + 8 - 5\sin^2 2x = 8$

$\Rightarrow \cos 4x = 0 \Rightarrow 4x = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\pi}{8},\ n \in I$

$\therefore x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \ldots\ldots\ldots\ldots, \frac{15\pi}{8}.$

$\therefore$ Number of solutions is 8. **Ans.**

47. Let the curve C be the mirror image of the parabola $y^2 = 4x$ with respect to the line $x + y + 4 = 0$. If A and B are the points of intersection of C with the line $y = -5$, then the distance between A and B is

Ans. **[4]**

Sol. Let image of $(t^2, 2t)$ in line $x + y + 4 = 0$ be (h, k).

$\therefore \frac{h-t^2}{1} = \frac{k-2t}{1} = \frac{-2(t^2+2t+4)}{1^2+1^2}$

$\Rightarrow h - t^2 = k - 2t = -t^2 - 2t - 4$

$\Rightarrow h = -2t - 4$ and $k = -t^2 - 4$

$\Rightarrow t = -\left(\frac{h+4}{2}\right)$

$\therefore k + 4 = -\left(\frac{h+4}{2}\right)^2$

$\Rightarrow (h+4)^2 = -4(k+4)$

$\therefore$ Locus is (curve C)

$\therefore (x + 4)^2 = - 4 (y + 4)$

$\therefore$ For point of intersection with $y = - 5$

$(x + 4)^2 = - 4 (- 5 + 4) = 4$

$\Rightarrow x + 4 = \pm 2 \Rightarrow x = - 2$ or $- 6$

$\therefore$ Distance AB will be 4. **Ans.**

48. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96 is

Ans. 8

Sol. Let minimum number of tosses be n.

$\therefore$ Probability of atleast two heads = 1 – P (no head) – P (exactly one head)

$$\Rightarrow 1 - \left(\frac{1}{2}\right)^n - {}^nC_1 \cdot \left(\frac{1}{2}\right)^{n-1} \cdot \left(\frac{1}{2}\right) \geq 0.96$$

$$\Rightarrow 1 - (n + 1) \left(\frac{1}{2}\right)^n \geq 0.96$$

$$\Rightarrow (0.04) \geq \frac{(n+1)}{2^n}$$

$$\Rightarrow 2^{n+2} \geq 100 (n + 1)$$

By hit and trial

$\therefore$ Minimum value of n will be 8. **Ans.**

Section 2

(Maximum Marks : 40)

- This section contains TEN questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- Marking Scheme:

+ 4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.

0 If none of the bubbles is darkened.

– 2 In all other cases.

49. In R^3, consider the planes P_1: $y = 0$ and P_2: $x + z = 1$. Let P_3 be a plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0, 1, 0) from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relations is(are) true?

(A) $2\alpha + \beta + 2\gamma + 2 = 0$ (B) $2\alpha - \beta + 2\gamma + 4 = 0$

(C) $2\alpha + \beta - 2\gamma - 10 = 0$ (D) $2\alpha - \beta + 2\gamma - 8 = 0$

Ans. **[B, D]**

Sol. Equation of plane P_3 will be $(x + z - 1) + \lambda y = 0$

$\Rightarrow x + \lambda y + z - 1 = 0$

Distance from (0, 1, 0) is equal to 1

$$\Rightarrow \frac{|0+\lambda \cdot 1+0-1|}{\sqrt{1+\lambda^2+1}} = 1$$

$\Rightarrow (\lambda - 1)^2 = \lambda^2 + 2 \Rightarrow \lambda^2 - 2\lambda + 1 = \lambda^2 + 2$

$\Rightarrow 2\lambda = -1 \Rightarrow \lambda = \frac{-1}{2}.$

$\therefore$ Plane will be $x - \frac{y}{2} + z - 1 = 0 \Rightarrow 2x - y + 2z - 2 = 0$

Distance from $(\alpha, \beta, \gamma) = 2$

$$\Rightarrow \frac{|2\alpha-\beta+2\gamma-2|}{\sqrt{2^2+1^2+2^2}} = 2 \Rightarrow 2\alpha - \beta + 2\gamma - 2 = \pm 6$$

$\Rightarrow 2\alpha - \beta + 2\gamma = 8$ or $2\alpha - \beta + 2\gamma = -4.$

50. In R^3, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes P_1: $x + 2y - z + 1 = 0$ and P_2 : $2x - y + z - 1 = 0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M?

(A) $\left(0, \frac{-5}{6}, \frac{-2}{3}\right)$ (B) $\left(\frac{-1}{6}, \frac{-1}{3}, \frac{1}{6}\right)$ (C) $\left(\frac{-5}{6}, 0, \frac{1}{6}\right)$ (D) $\left(\frac{-1}{3}, 0, \frac{2}{3}\right)$

Ans. **[A, B]**

Sol. $\because$ Line is at constant distance from both the planes

$\therefore$ Line will be parallel to both planes

$\therefore$ perpendicular to their normals

Normal will be $\vec{n_1} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{n_2} = 2\hat{i} - \hat{j} + \hat{k}$

$$\therefore \vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{vmatrix} = \hat{i} - 3\hat{j} - 5\hat{k}$$

$\therefore$ Equation of line will be $= \frac{x-0}{1} = \frac{y-0}{-3} = \frac{z-0}{-5}$

Let foot of perpendicular from (0, 0, 0) on plane P_1: $x + 2y - z + 1 = 0$ be (α, β, γ)

$$\therefore \frac{\alpha-0}{1} = \frac{\beta-0}{2} = \frac{\gamma-0}{-1} = \frac{-(0+0+0+1)}{1^2+2^2+(-1)^2} = \frac{-1}{6}$$

$\therefore \alpha = \frac{-1}{6}, \beta = \frac{-2}{6} \text{ and } \gamma = \frac{1}{6}$

Locus of feet of perpendicular drawn from line upon plane will be a parallel line passing through $\left(\frac{-1}{3}, \frac{2}{3}, \frac{1}{3}\right)$ = (α, β, γ)

$\therefore$ ne will be Li $\frac{x+\frac{1}{6}}{1} = \frac{y+\frac{2}{6}}{-3} = \frac{z-\frac{1}{6}}{-5} = \lambda$

(A) If x = 0

$\therefore y + \frac{2}{6} = \frac{-3}{6} \Rightarrow y = \frac{-5}{6} \text{ and} - \frac{1}{6} = \frac{-5}{6} \Rightarrow \frac{-4}{6} = \frac{-2}{3}$

$\therefore$ Point is $\left(0, \frac{-5}{6}, \frac{-2}{3}\right)$.

(B) If $x = \frac{-1}{6}, \text{then} = \frac{-2}{6} \text{ and } z = \frac{1}{6}$

(C) & (D)

If y = 0, then $\frac{x+\frac{1}{6}}{1} = \frac{0+\frac{2}{6}}{-3} = \frac{z-\frac{1}{6}}{-5}$

$\Rightarrow x + \frac{1}{6} = \frac{-1}{9} \Rightarrow x = -\frac{1}{6} - \frac{1}{9} = \frac{-3-2}{18} = \frac{-5}{18}$

$\Rightarrow \frac{1}{6} = \frac{5}{9} \Rightarrow z = \frac{5}{9} + \frac{1}{6} = \frac{13}{18}.$

$\Rightarrow$ Answer is (A) & (B).

51. Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$ then which of the following is(are) the coordinates of P?

(A) $\left(4, 2\sqrt{2}\right)$ (B) $\left(9, 3\sqrt{2}\right)$ (C) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ (D) $\left(1, \sqrt{2}\right)$

Ans. **[A, D]**

Sol. $m_{OP} = \frac{2}{t_1}; \ m_{OQ} = \frac{2}{t_2}$

$m_{OP} \cdot m_{OQ} = -1 \Rightarrow \frac{2}{t_1} \cdot \frac{2}{t_2} = -1 \Rightarrow t_1 t_2 = -4$

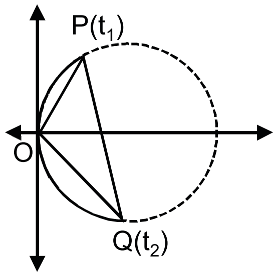
Area (ΔOPQ) = 3

$$\frac{1}{2}\begin{vmatrix} at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm 3\sqrt{3}$$

$$2a^2\left(t_1^2t_2 - t_1t_2^2\right) = \pm 6\sqrt{2}$$

$t_1t_2\,(t_1 - t_2) = \pm 12\sqrt{2}$

$\Rightarrow t_1 - t_2 = \pm 3\sqrt{2}$ ....(1)



$\Rightarrow (t_1 + t_2)^2 = (t_1 - t_2)^2 + 4t_1t_2 = 18 - 16 = 2$

$\Rightarrow t_1 + t_2 = \pm\sqrt{2}$ ....(2)

From equation (1) & (2)

$\Rightarrow t_1 = 2\sqrt{2}$ or $\sqrt{2}$

$\therefore$ Coordinates of P are $\left(4,\ 2\sqrt{2}\right)$ or $\left(1,\ \sqrt{2}\right)$.

52. Let y (x) be a solution of the differential equation $(1 + e^x)y' + ye^x = 1$. If y (0) = 2, then which of the following statements is(are) true?

(A) y (– 4) = 0

(B) y (– 2) = 0

(C) y (x) has a critical point in the interval (– 1, 0)

(D) y (x) has no critical point in the interval (– 1, 0)

Ans. **[A, C]**

Sol. $(1 + e^x)\,y' + y\,e^x = 1$

$$y' + \left(\frac{e^x}{1+e^x}\right)y = \frac{1}{1+e^x}$$

$$\text{I.F.} = e^{\int \frac{e^x}{1+e^x}dx} = e^{\ln(1+e^x)} = 1 + e^x.$$

$$y\,(1 + ex) = \int 1\cdot dx + C$$

$y\,(1 + e^x) = x + C$

Curve is passing through point (0, 2)

$\therefore$ C = 4

$$\therefore y\,(1 + e^x) = x + 4 \Rightarrow y = \frac{x+4}{1+e^x}$$

$y(-4) = 0 \Rightarrow$ (A)

$$\frac{dy}{dx} = \frac{(1+e^x)\cdot 1 - (x+4)e^x}{(1+e^x)^2} = 0$$

$\Rightarrow \quad 1 + e^x - (x + 4)e^x = 0$

$\Rightarrow \quad e^{-x} + 1 = x + 4$

$e^{-x} = x + 3$

$e^{-x} - x - 3 = 0$

Let $g(x) = e^{-x} - x - 3$

g (0) = 1 – 3 (– ve)

g (– 1) = e^1 + 1 – 3 (+ve)

∴ $e^{-x} - x - 3 = 0$ has a solution in (–1, 0)

53. Consider the family of all circles whose centers lie on the straight line y = x. If this family of circles is represented by the differential equation Py" + Qy' + 1 = 0, where P, Q are functions of x, y and y' $\left(\text{here } y' = \frac{dy}{dx}, y'' = \frac{d^2y}{dx^2}\right)$, then which of the following statement(s) is(are) true?

(A) P = y + x

(B) P = y – x

(C) P + Q = 1 – x + y + y' + $(y')^2$

(D) P – Q = x + y – y' – $(y')^2$.

Ans. **[B, C]**

Sol. $(x-a)^2 + (y-a)^2 = r^2$

$2(x-a) + 2(y-a)y' = 0$

$1 + (y-a)y'' + (y')^2 = 0$

$$1 + \left(y - \frac{x+yy'}{1+y'}\right)y'' + (y')^2 = 0$$

$$1 + y' + \left(y(1+y') - (x+yy')\right)y'' + (y')^2(1+y') = 0$$

$$1 + (y + yy' - x - yy')y'' + y'(1+y'(1+y')) = 0$$

P = y – x, Q = 1 + y' + $(y')^2$

P + Q = y – x + 1 + y' + $(y')^2$

54. Let g: R → R be a differentiable function with g (0) = 0, g ' (0) = 0 and g ' (1) ≠ 0.

$$\text{Let } f(x) = \begin{cases} \frac{x}{|x|}g(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

and h (x) = $e^{|x|}$ for all x ∈ R. Let (f o h)(x) denote $f(h(x))$ and (h o f) (x) denote $h(f(x))$.

Then which of the following is(are) true?

(A) f is differentiable at x = 0

(B) h is differentiable at x = 0

(C) f o h is differentiable at x = 0

(D) h o f is differentiable at x = 0

Ans. [A, D]

Sol.

(A) $$f'(0^+) = \lim_{h\to 0}\frac{\frac{h}{h}g(h) - 0}{h} = g'(0) = 0$$

$$f'(0^-) = \lim_{h\to 0}\frac{\frac{h}{h}g(-h) - 0}{h} = -g'(0) = 0$$

∴ f (x) is divisible at x = 0

(B) Obviously h (x) is non-derivable at x = 0

(C) $f \circ h\ (x) = \frac{e^{|x|}}{|e^{|x|}|} g\left(e^{|x|}\right) = g\left(e^{|x|}\right)$

$f \circ h\ (0+) = \lim_{h \to 0} \frac{g(e^h) - g(1)}{h} = \lim_{h \to 0} \frac{g(e^h) - g(1)}{h} = g'(1)$

$f \circ h\ (0h) = \lim_{h \to 0} \frac{g(e^h) - g(1)}{-h} = -g'(1)$

(D) $h \circ f\ (x) = e^{\left|\frac{x}{|x|} g(x)\right|} = e^{|g(x)|}$, since g ' (0) = 0 and g (0) = 0

$\Rightarrow$ | g (x) | is differentiable is equal to zero.

$\Rightarrow$ h o f (x) is derivable at x = 0.

55. Let f(x) = $\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x \in R$ and g(x) = $\frac{\pi}{2}$ sin x for all $x \in R$. Let (fog)(x) denote $f\left(g(x)\right)$ and (g o f)(x) denote $g\left(f(x)\right)$. Then which of the following is (are) true?

(A) Range of f is $\left[\frac{-1}{2}, \frac{1}{2}\right]$

(B) Range of f o g is $\left[\frac{-1}{2}, \frac{1}{2}\right]$

(C) $\lim_{x \to 0} \frac{f(x)}{g(x)} = \frac{\pi}{6}$

(D) There is an $x \in R$ such that (g o f)(x) = 1

Ans. **[A, B, C]**

Sol.

(A) Range of f (x) is $\left[\frac{-1}{2}, \frac{1}{2}\right]$

(B) Range of g (x) is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

f o g (x) = f [g (x)] = $\left[\frac{-1}{2}, \frac{1}{2}\right]$

(C) $\lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)}{\frac{\pi}{2}\sin x} = \lim_{x \to 0} \frac{\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)}{\frac{\pi}{2}\sin x} = \frac{\pi}{6}$

(D) $g \circ f\ (x) = g\left(f(x)\right) = \frac{\pi}{2}\sin\left(f(x)\right)$

$\because$ f (x) $\in \left[\frac{-1}{2}, \frac{1}{2}\right]$ = sin f (x) < $\frac{1}{2}$

$\therefore$ $g\left(f(x)\right)$ can not be equal to for any $x \in R$.

56. Let ΔPQR be a triangle. Let $\vec{a} = \overrightarrow{QR}$, $\vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If $|\vec{a}| = 12, |\vec{b}| = 4\sqrt{3}$ and $\vec{b}\,.\,\vec{c} = 24$, then which of the following is (are) true?

(A) $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$

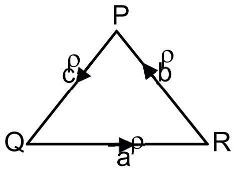
(B) $\frac{|\vec{c}|^2}{2} + |\vec{a}| = 30$

(C) $|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 48\sqrt{3}$

(D) $\vec{a} \cdot \vec{b} = -72$

Ans. A, C, D

Sol.



$|\vec{a}|$ = QR = 12

$|\vec{b}|\, PR = 4\sqrt{3}$

$\vec{b} \cdot \vec{c} = 24$

$\vec{b} \cdot \vec{c} = |b||\vec{c}| \cos(\pi - p) = 24$

$-4\sqrt{3}$ (cos p) c = 24

$c \cos p = \frac{-6}{\sqrt{3}}$ ......(1)

$\cos p = \frac{b^2 + c^2 - a^2}{2bc}$

$\frac{-6}{\sqrt{3}\,c} = \frac{48 + c^2 - a^2}{8\sqrt{3}\ c}$

$\frac{c^2 - a^2 + 48}{8} = -6$

$c^2 - a^2 = -96$

$c^2 - 144 = -96$

$c^2 = 144 - 96$

$c^2 = 48$

$c = 4\sqrt{3}$ .....(2)

From eqn. (1) and (2), we get

$\cos p = \frac{-6}{12} = \frac{-1}{2}$; $\angle P = 120°$

(A) $\frac{|\vec{c}|^2}{2} - |\vec{a}| = \frac{48}{2} - 12 = 12$

(C) $\left|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}\right|$

$\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{a} \times \vec{b} = \vec{c} \times \vec{a} = \vec{b} \times \vec{c} \Rightarrow |\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 2|\vec{b} \times \vec{c}|$

$= 2\left|\vec{b}\right|\left|\vec{c}\right| \sin 120° = 48\sqrt{3}$

(D) $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{a} \cdot \vec{b} + b^2 + 24 = 0$

$\vec{a} \cdot \vec{b} = -48 - 24 = -72$

57. Let X and Y be two arbitrary, 3 × 3, non-zero, skew symmetric matrices and Z be an arbitrary 3 × 3, non-zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric?

(A) $Y^3Z^4 - Z^4Y^3$ (B) $X^{44} + Y^{44}$ (C) $X^4Z^3 - Z^3X^4$ (D) $X^{23} + Y^{23}$

Ans. **[C, D]**

Sol.

(A) $(y^3z^4 - z^4z^3)^T = y^3z^4 - z^4y^3$ (symmetric)

(B) $(x^{44} + y^{44})^T = x^{44} + y^{44}$ (symmetric)

(C) $(x^4z^3 - z^3x^4)^T = -x^4z^3 + z^3\,x^4$(skew)

(D) $(x^{23} + y^{23})^T = -x^{23} - y^{23}$ (skew)

58. Which of the following values of α satisfy the equation $\begin{vmatrix}(1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2\\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2\\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2\end{vmatrix} = -648\alpha$?

(A) – 4 (B) 9 (C) – 9 (D) 4

Ans. **[B, C]**

Sol. $$\begin{vmatrix}1 & 1 & 1\\ 4 & 2 & 1\\ 9 & 3 & 1\end{vmatrix}\begin{vmatrix}1 & 2\alpha & \alpha^2\\ 1 & 4\alpha & 4\alpha^2\\ 1 & 6\alpha & 9\alpha^2\end{vmatrix} = -\alpha 3\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 4\\ 1 & 3 & 9\end{vmatrix}^2 = -\alpha 3\begin{vmatrix}1 & 1 & 1\\ 0 & 1 & 3\\ 0 & 1 & 5\end{vmatrix}^2$$

$\Rightarrow 4\alpha^3 = -648\alpha \Rightarrow \alpha = 0,\ \alpha^2 = 81 \Rightarrow \alpha = \pm 9$. **Ans.**

Section 3

- This section contains TWO questions.
- Each question contains two columns, Column-I and Column-II
- Column-I has four entries (A), (B), (C) and (D)
- Column-II has five entries (P), (Q), (R), (S) and (T)
- Match the entries in Column-I with the entries in Column-II
- One or more entries in Column-I may match with one or more entries in Column-II
- The ORS contains a 4 × 5 matrix whose layout will be similar to the one shown below:

(A)	P	Q	R	S	T
(B)	P	Q	R	S	T
(C)	P	Q	R	S	T
(D)	P	Q	R	S	T

- For each entry in Column-I, darken the bubbles of all matching entries. For example, if entry (A) in Column-I matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).
- Marking Scheme:

+ 2 If only the bubble(s) corresponding to all the correct match(es) is(are) darkened.

0 If none of the bubbles is darkened.

– 1 In all other cases.

59. **Column-I** **Column-II**

(A) In R^2, if the magnitude of the projection vector of the vector $\alpha\hat{i}+\beta\hat{j}$ on $\sqrt{3}\hat{i}+\hat{j}$ is $\sqrt{3}$ is and if $\alpha = 2 + \sqrt{3}\beta$, then possible value(s) of $|\alpha|$ is (are) (P) 1

(B) Let a and b be real numbers such that the function (Q) 2

$$f(x) = \begin{cases} -3ax^2-2, & x<1 \\ bx+a^2, & x\geq 1 \end{cases}$$

is differentiable for all $x \in R$. Then possible value(s) of a is (are)

(C) Let $\omega \neq 1$ be a complex cube root of unity. (R) 3

If $(3-3\omega+2\omega^2)^{4n+3} + (2+3\omega-3\omega^2)^{4n+3} + (-3+2\omega+3\omega^2)^{4n+3} = 0$,

then possible value(s) of n is (are)

(D) Let the harmonic mean of two positive real numbers a and b be 4. (S) 4

If q is a positive real number such that a, 5, q, b is an arithmetic progression, then the value(s) of $|q-a|$ is (are)

(T) 5

Ans. **[(A) P, Q ; (B) P, Q ; (C) P, Q, S, T ; (D) Q, T]**

Sol.

(A) $$\sqrt{3} = \frac{\alpha\sqrt{3}+\beta}{2\sqrt{\alpha^2+\beta^2}}\cdot\sqrt{\alpha^2+\beta^2}$$

$\alpha\sqrt{3}+\beta = \pm 2\sqrt{3}$ ....(1)

Given, $\alpha - \beta = \sqrt{3}\,2$ ....(2)

$\beta = 0$, $\alpha = 2 \Rightarrow$ (P) & (Q)

(B) f(x) is continuous at x = 1

$f(1) = \lim_{x\to 1^-} f(x)$

$b + a^2 = -3a - 2$

$b + a^2 + 3a + 2 = 0$ ....(1)

Differentiable at x = 1

$$\therefore \text{R.H.D.} = \lim_{h\to 0}\frac{b(1+h)+a^2-b-a^2}{h} = \lim_{h\to 0}\frac{-3a(1-h)^2-2-b-a^2}{-h} = \text{L.H.D.}$$

$$\lim_{h\to 0}\frac{bh}{h} = \lim_{h\to 0}\frac{-3a+6ah-3ah^2-2-b-a^2}{-h} = \lim_{h\to 0}\frac{6ah-3ah^2-(b+a^2+3a+2)}{-h}$$

$\Rightarrow \quad \lim_{h\to 0}\frac{bh}{h} = \lim_{h\to 0}\frac{6ah - 3ah^2}{-h}$ [From equation (1)]

By equation

b = – 6a

Equation (1)

$-6a + a^2 + 3a + 2 = 0 \Rightarrow a^2 - 3a + 2 = 0 \Rightarrow a^2 - 3a + 2 = 0$

$\Rightarrow (a - 1)(a - 2) = 0 \Rightarrow a = 1, a = 2 \Rightarrow$ (P) & (Q)

(C) $(3 - 3\omega + 2\omega^2)^{4n+3} + (2 + 3\omega - 3\omega^2)^{4n+3} + (-3 + 2\omega + 3\omega^2)^{4n+3} = 0$

(I) $3 - 3\omega + 2\omega^2 = 1 - 5\omega$

(II) $2 + 3\omega - 3\omega^2 = \omega(1 - 5\omega)$

(III) $-3 + 2\omega + 3\omega^2 = \omega^2(1 - 5\omega)$

Now, $(1 - 5\omega)^{4n+3}(1 + \omega^{4n+3} + (\omega^2)^{4n+3}) = 0$

$1 + \omega^{3(n+1)+n} + \omega^{3(2n+2)+2n} = 0$

$1 + \omega^n + \omega^{2n} = 0$

at n = 1, 2, 4, 5.

(D) $\frac{2ab}{a+b} = 4$ ....(1)

a, 5, q, b

$d = 5 - a;\ q = b - 5 + a \Rightarrow q - a = b - 5 \Rightarrow d = \frac{b-a}{3}$

$a + d = 5 \Rightarrow a - \frac{b-a}{3} = 5 \Rightarrow b = 15 - 2a$

$\Rightarrow \frac{2ab}{a+b} = 4 \Rightarrow \frac{2a(15-2a)}{a+15-2a} = 4$

$a = 6,\ a = \frac{5}{2},\ b = 3,\ b = 10$

$|q - a| = |b - 5| = 5, 2 \Rightarrow$ (Q) & (T).

60.

	Column-I	Column-II	
(A)	In a triangle ΔXYZ, let a, b and c be the lengths of the sides opposite to the angles X, Y and Z respectively. If $2(a^2 - b^2) = c^2$ and $\lambda = \frac{\sin(X-Y)}{\sin Z}$, then possible values of n for which $\cos(n\pi\lambda) = 0$ is (are)	(P)	1
(B)	In a triangle ΔXYZ, let a, b and c be the lengths of the sides opposite to the angles X, Y and Z respectively.	(Q)	2

If 1 + cos 2X – 2cos 2Y = 2sin X sin Y, then possible value(s) of $\frac{a}{b}$ is (are)

(C) In R^2, let $\sqrt{3}\hat{i}+\hat{j}, \hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ be the position vectors of X, Y and Z with respect to the origin O, respectively. If the distance of Z from the bisector of the acute angle of $\overrightarrow{OX}$ with $\overrightarrow{OY}$ is $\frac{3}{\sqrt{2}}$, with is, then possible value(s) of | β | is (are) (R) 3

(D) Suppose that F(α) denotes the area of the region bounded by x = 0, x = 2, y^2 = 4x and y = | αx – 1 | + | αx – 2 | + αx, where α ∈ {0, 1}. (S) 5

Then the value(s) of F(α) + $\frac{8\sqrt{2}}{3}$, when α = 0 and α = 1, is (are) (T) 6

Ans. **[(A) P, R, S; (B) P; (C) P, Q; (D) S, T]**

Sol.

(A) 2 ($\sin^2x - \sin^2y$) = $\sin^2z$

2sin (x – y) = sin z

$\lambda = \frac{\sin(x-y)}{\sin z} = \frac{1}{2}$

cos (nπλ) = 0

nπλ = (2p + 1) $\Rightarrow \frac{\pi}{2} \Rightarrow \frac{n\pi}{2}$ = (2p + 1) $\frac{\pi}{2}$ ⇒ n = (2p + 1) ⇒ (P), (Q) & (S)

(B) 1 + cos 2x – cos 2y – cos 2y = 2 – sin x sin y

$2\sin^2y$ + 2sin (x + y) sin (y – x) = 2sin x sin y

$\sin^2y + \sin^2y - \sin^2x$ = sin x sin y

$2\sin^2y - \sin^2x$ = sin x sin y

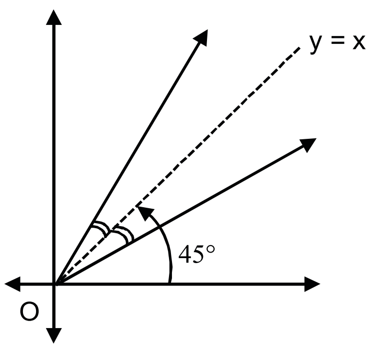
$2b^2 - a^2 = ab$

a = b or a = – 2b (rejected)

$\frac{a}{b} = 1 \Rightarrow (P)$

(C) $\frac{|\beta-(1-\beta)|}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

2β – 1 = – 3, 3



2β = – 2, 4

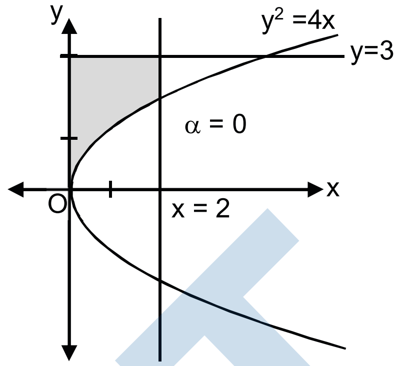
$\beta = -1, 2$

$|\beta| = 1, 2 \qquad \Rightarrow \qquad$ P, Q

(D) If $\alpha = 0$

$$\int_0^2 (3 - 2\sqrt{x})\, dx = \left(3x - 2x^{\frac{3}{2}} \cdot \frac{2}{3}\right)_0^2$$

$$F(\alpha) = 6 - 2 \cdot 2\sqrt{2} \cdot \frac{2}{3}$$



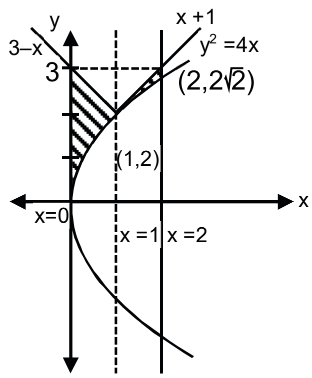
$$F(a) + \frac{8\sqrt{2}}{3} = 6.$$ **Ans.**

If $\alpha = 1$

$y = |x - 1| + |x - 2| + x$

$x \in (-\infty, 1)$	$x \in [1, 2)$	$x \in [2, \infty)$
$y = 1 - x - x + 2 + x$	$y = x - 1 - x + 2 + x$	$y = x - 1 + x - 2 + x$
$= 3 - x$	$y = x + 1$	$y = 3x - 3 = 3(x - 1)$

$$A = \int_0^1 \left((3 - x) - (2\sqrt{x})\right) dx + \int_1^2 \left((x + 1) - (2\sqrt{x})\right) dx$$



$$= \left(3x - \frac{x^2}{2} - 2\cdot\frac{2}{3}\cdot x^{\frac{3}{2}}\right)_0^1 + \left(\frac{x^2}{2} + x - 2\cdot\frac{2}{3}\cdot x^{\frac{3}{2}}\right)_1^2$$

$$= \left(3 - \frac{1}{2} - \frac{4}{3}\right) + \left(2 + 2 - \frac{8}{3}\sqrt{2}\right) + \left(\frac{1}{2} + 1 - \frac{4}{3}\right)$$

$$F(\alpha) = 3 - \frac{1}{2} + 4 - \frac{8}{3}\sqrt{2} - \frac{3}{2} = 5 - \frac{8}{3}\sqrt{2}$$

$$F(a) + \frac{8}{3}\sqrt{2} = 5$$ **Ans.**